

Gravastars with higher dimensional spacetimes

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Abstract We present a new model of gravastar in the higher dimensional Einstein-Maxwell spacetime including Einstein's cosmological constant Λ . Following Mazur and Mottola [1,2] we obtain a set of solutions for gravastar. This gravastar is described by three different regions namely, (I) Interior region, (II) Intermediate thin spherical shell and (III) Exterior region. The pressure within the interior region is equal to the negative matter density which provides a repulsive force over the shell. This thin shell is formed by ultra relativistic plasma, where the pressure is directly proportional to the matter-energy density which does counter balance the repulsive force from the interior whereas the exterior region is completely vacuum assumed to be de Sitter spacetime which can be described by the generalized Schwarzschild solution. With this specification we find out a set of exact and non-singular solutions of the gravastar which seems physically very interesting and reasonable.

Keywords General relativity; Gravastar; Dark Energy

1 Introduction

In general relativity of Einstein there is an inherent feature of singularity at the end point of gravitationally collapsing system and has been remains an embarrassing situation to the astrophysical community. To overcome this odd phase of a stellar body where all the physical laws break down, Mazur and Mottola [1,2] proposed a new model considering the gravitationally vacuum star which was termed in brevity as Gravastar, that brings up a new arena in the gravitational system. They generated a new type of solution to this system of gravitational collapse by extending the idea of Bose-Einstein condensation by constructing gravastar as a cold, dark and compact object of interior de Sitter condensate phase surrounded by a thin shell of ultra relativistic matter whereas the exterior region is completely vacuum, i.e. the Schwarzschild spacetime is at the outside. The shell is very thin

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but of finite width in the range $r_1 < r < r_2$, where r_1 and r_2 are the interior and exterior radii of the gravastar. With this unique specification we can divide the entire system of gravastar into three specific segments based on the equation of state (EOS) as follows: (I) Interior: $0 \leq r < r_1$, with EOS $p = -\rho$, (II) Shell: $r_1 \leq r \leq r_2$, with EOS $p = +\rho$, and (III) Exterior: $r_2 < r$, with EOS $p = \rho = 0$.

The abovementioned model of gravastar has been studied by researchers which opened up a new challenges in the gravitational research to obtain a singularity free solution of the Einstein field equations. Therefore, it is supposed to be an alternative solution of black hole and has been studied by several authors in different context of astrophysical systems [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

The negative matter density in the interior region creates a repulsive pressure acting radially outward from the centre of the gravastar (*i.e.* $r = 0$) over the shell whereas the shell of positive matter density provides the necessary gravitational pull to balance this repulsive force within the interior. It is assumed that the dark energy (or the vacuum energy) is responsible for this repulsive pressure from the interior. In a general consideration, the EOS $p = -\rho$ is suggesting that the repulsive pressure is an agent, responsible for accelerating phase of the present universe and is known as the Λ -dark energy [21, 22, 23, 24, 25]. In literature this EOS is termed as a ‘false vacuum’, ‘degenerate vacuum’, or ‘ ρ -vacuum’ [26, 27, 28, 29]. Therefore, in this context one can note that gravastar may have some connection to the dark star [30, 31, 32, 16].

The EOS for the shell $p = \rho$ represents essentially a stiff fluid model as conceived by Zel’dovich [33] in connection to the cold baryonic universe. The idea has been considered by several scientists for various situations in cosmology [34, 35] as well as astrophysics [36, 37, 38].

Einstein introduced cosmological constant Λ in his field equations to make it consistent with the Mach principle to obtain a static and non-expanding solutions of the universe without having any valid physical interpretation of the proposed model. In his model the constant Λ with the right sign could produce a repulsive pressure to exactly counter balance the gravitational attraction and hence could keep the model of the universe static.

But after the experimental verification of expanding universe by Edwin Hubble between 1922 to 1924 [39] and the success of FLRW cosmology made Einstein realize that the universe has been expanding with an acceleration. That is why later on Einstein discarded the cosmological constant from his field equation. However, though it is abandoned by Einstein but for the physical requirement to describe one-loop quantum vacuum fluctuations, the Casimir effect [40], cosmological constant had to appear once again in the theory with a form as $T_{ij} = \Lambda g_{ij}/8\pi G$, where T_{ij} and g_{ij} are the stress energy tensor and the metric tensor respectively and G is the usual Newtonian constant.

Recent observations conducted by WMAP suggests that 73% of the total mass-energy of the universe is dark energy [41, 42]. It is believed that this dark energy plays an important role for the evolution of the universe and in order to describe the dark energy scientists have recall the erstwhile cosmological constant. Therefore, in the modern cosmology this cosmological constant is treated as a strong candidate for the dark energy which is responsible for the accelerating phase of the present universe.

Very recently a charged gravastar in higher dimension has proposed by Bhar [43] admitting conformal motion and also by Ghosh et al. [20] without admitting the conformal motion in the framework of Mazur and Mottola model. These works provide an alternative solution to the static black holes. Usmani et al. [16] have also found solution of neutral gravastar in higher dimension without admitting the conformal motion. The present study on gravastar basically is an extension of the work of Usmani et al. [16] as mentioned above and its generalization to the higher dimensional spacetime in presence of the cosmological constant (Λ). Therefore, the main motivation of this work is to study the effects of the cosmological constant for construction of gravastars and also to study the higher dimensional effects, if any.

The present investigations are based on the plans as follows: The background of the model has been implemented by Einstein-Maxwell geometry discuss in Sect. 2, whereas the he solution of interior spacetime, the thin shell and exterior space-time of the gravastar has been discussed in Sect. 3. Then we have discussed the junction conditions for the different regions of the gravatar in Sect. 4. In the Sect. 5 we explore some physical features of the model, viz. proper length, Energy, Entropy and also study their variation with the radial parameter which followed by the discussion and concluding remarks at the end in Sect. 6.

2 The Einstein-Maxwell spacetime geometry

The Einstein-Hilbert action coupled to matter is given by

$$I = \int d^D x \sqrt{-g} \left(\frac{R_D}{16\pi G_D} + L_m \right), \quad (1)$$

where the curvature scalar in D -dimensional spacetime is represented by R_D , with G_D is the D -dimensional Newtonian constant and L_m denotes the Lagrangian for the matter distribution. We obtain the following Einstein equation by varying the above action with respect to the metric

$$G_{ij}^D = -8\pi G_D T_{ij}, \quad (2)$$

where G_{ij}^D denotes the Einstein's tensor in D -dimensional spacetime.

The interior of the star is assumed to be perfect fluid type and can be given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij}, \quad (3)$$

where ρ represents the energy density, p is the isotropic pressure, and u^i is the D -velocity of the fluid.

Here in the present study it is assumed that the gravastars in higher dimensions have the D -dimensional spacetime with the structure $R^1 X S^1 X S^d (d = D - 2)$, where the range of the radial coordinate is S^1 and the time axis is represented by R^1 . For this purpose, we consider a static spherically symmetric metric in $D = d + 2$ dimension as

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega_d^2, \quad (4)$$

where $d\Omega_d^2$ is the linear element of a d -dimensional unit sphere, parameterized by the angles $\phi_1, \phi_2, \dots, \phi_d$ as follows:

$$d\Omega_d^2 = d\phi_d^2 + \sin_2 \phi_d [d\phi_{d-1}^2 + \sin_2 \phi_{d-1} \{d\phi_{d-2}^2 + \dots + \sin_2 \phi_3 (d\phi_2^2 + \sin_2 \phi_2 d\phi_1^2) \dots\}].$$

Now the Einstein field equations for the metric (4), together with the energy-momentum tensor given in Eq. (3) in presence of the non-zero cosmological constant Λ , yield

$$-e^{-\lambda} \left[\frac{d(d-1)}{2r^2} - \frac{d\lambda'}{2r} \right] + \frac{d(d-1)}{2r^2} = 8\pi G_D \rho + \Lambda, \quad (5)$$

$$e^{-\lambda} \left[\frac{d(d-1)}{2r^2} + \frac{d\nu'}{2r} \right] - \frac{d(d-1)}{2r^2} = 8\pi G_D p - \Lambda, \quad (6)$$

$$\frac{e^{-\lambda}}{2} \left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{\nu'^2}{2} - \frac{(d-1)(\lambda' - \nu')}{r} + \frac{(d-1)(d-2)}{r^2} \right] - \frac{(d-1)(d-2)}{2r^2} = 8\pi G_D p - \Lambda, \quad (7)$$

where ' ν ' denotes differentiation with respect to the radial parameter r . Here we have assumed $c = 1$ in geometrical unit.

In general relativity the conservation of energy-momentum is expressed with the aid of a stress-energy-momentum pseudotensor, i.e. $T^{ij}{}_{;j} = 0$ and can be expressed in its general form with D -dimension as

$$\frac{1}{2}(\rho + p)\nu' + p' = 0. \quad (8)$$

In the next Sect. 3 we shall formulate special explicit forms of the energy conservation equations for all the three regions, viz. interior, intermediate thin shell and exterior spacetimes.

3 The gravastar models

3.1 Interior spacetime

In the interior region of the gravastar it is assumed that the negative pressure is acting radially outward from the center of the spherically symmetric system to balance the inward pull from the shell. Following Mazur-Mottola [1], the EOS for the interior region can be provided in the form

$$p = -\rho. \quad (9)$$

Using Eq. (8) and the above EOS (9), we obtain

$$p = -\rho = \rho_c, \quad (10)$$

where ρ_c is the critical density of the interior region. Using Eq. (9) in the field equation (5), one obtains the solution of λ as

$$e^{-\lambda} = 1 - \frac{16\pi G_D \rho_c}{d(d+1)} r^2 - \frac{2Ar^2}{d(d+1)} + C_1 r^{1-d}, \quad (11)$$

where C_1 is an integration constant. Since $d \geq 2$ for dimension higher than three and the solution is regular at $r = 0$, so we demand for $C_1 = 0$. Thus essentially we get

$$e^{-\lambda} = 1 - \frac{16\pi G_D \rho_c}{d(d+1)} r^2 - \frac{2Ar^2}{d(d+1)}. \quad (12)$$

Using Eq. (9) one may obtain from Eqs. (5) and (6), the following relation

$$\ln k = \lambda + \nu \Rightarrow e^\nu = k e^{-\lambda}, \quad (13)$$

where k is an integration constant.

Thus we have the following interior solutions for the metric potentials λ and ν as follows

$$k e^{-\lambda} = e^\nu = k \left[1 - (C_2 - C_3) r^2 \right], \quad (14)$$

where $C_2 = \frac{16\pi G_D \rho_c}{d(d+1)}$ and $C_3 = \frac{2\Lambda}{d(d+1)}$.

From Eq. (10) it is observed that the matter density remains constant over the entire interior spacetime. Thus we can calculate the active gravitational mass $M(r)$ in higher dimensions as

$$M(r) = \int_0^{r_1=R} \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+1}{2}\right)} \right] r^d \rho_c dr = \left[\frac{2\pi^{\frac{d+1}{2}} \rho_c}{(d+1)\Gamma\left(\frac{d+1}{2}\right)} \right] R^{d+1}, \quad (15)$$

where R is the internal radius of the gravastar. So the usual gravitational mass in for a d -dimensional sphere dimension can be represented by Eq. (15), which is directly proportional to the radius R and the matter density ρ .

In the interior region, therefore, the energy conservation equation (8) takes the special explicit form as follows:

$$p' = \frac{(p + \rho) \left[\frac{16\pi G_D \rho_c}{d(d+1)} - \frac{2\Lambda}{d(d+1)} \right] r}{1 - \left[\frac{16\pi G_D \rho_c}{d(d+1)} - \frac{2\Lambda}{d(d+1)} \right] r^2}. \quad (16)$$

3.2 Intermediate thin shell

Here we assume that the thin shell contains ultra-relativistic fluid of soft quanta and obeys the EOS

$$p = \rho. \quad (17)$$

It is difficult to obtain a general solution of the field equations in the non-vacuum region, i.e. within the shell. Therefore, we try to find an analytic solution within the thin shell limit, $0 < e^{-\lambda} \equiv h \ll 1$. To do so we set h to be zero to the leading order. Under this approximation, the field Eqs. (5) - (7) along with the above EOS, can be reformat in the following form

$$-\frac{h'}{2r} + \frac{(d-1)}{2r^2} = \frac{8\pi G_D \rho}{d} + \frac{\Lambda}{d}, \quad (18)$$

$$-\frac{(d-1)}{2r^2} = \frac{8\pi G_D p}{d} - \frac{\Lambda}{d}, \quad (19)$$

$$\frac{h'\nu'}{4} + \frac{(d-1)h'}{2r} - \frac{(d-1)(d-2)}{2r^2} = 8\pi G_D p - \Lambda. \quad (20)$$

Now using Eqs. (18) and (19), we find out an expression for h as

$$h = e^{-\lambda} = k_1 + 2(d-1) \ln r - \frac{2\Lambda r^2}{d}, \quad (21)$$

where k_1 is an integration constant. Here the range of r lies within the thickness of the shell, i.e. $r_1 = R, r_2 = R + \epsilon$, where $\epsilon \ll 1$ which is the thickness of the shell.

The other metric potential (ν) can be found as

$$e^{-\nu} = k_2 \frac{r^{2d}}{|2\Lambda r^2 - d^2 + d|}, \quad (22)$$

where k_2 is the integration constant.

Again using Eqs. (8) and (22) we get the pressure and matter density within the shell of the gravastar as

$$p = \rho = \rho_0 e^{-\nu} = \rho_0 k_2 \frac{r^{2d}}{|2\Lambda r^2 - d^2 + d|}. \quad (23)$$

In the intermidiate thin shell, therefore, the energy conservation equation (8) takes the special explicit form as follows:

$$p' = r(p + \rho) \left[\frac{2\Lambda}{2\Lambda r^2 - d^2 + d} - d \right]. \quad (24)$$

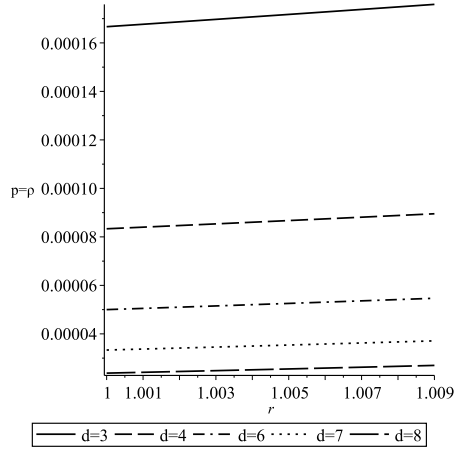


Fig. 1 Variation of the pressure and density of the ultra relativistic matter in the shell against r for different dimensions where the specific legends used are shown in the respective plots

3.3 Exterior spacetime

The EOS for the exterior region is defined as $p = \rho = 0$. In higher dimensions one can expect that the exterior solution is nothing but a generalization of the Schwarzschild solution. Now, following the work of Tangherlini [44] this can be obtained as

$$ds^2 = - \left(1 - \frac{\mu}{r^{d-1}} - \frac{2\Lambda r^2}{(d+1)d} \right) dt^2 + \left(1 - \frac{\mu}{r^{d-1}} - \frac{2\Lambda r^2}{(d+1)d} \right)^{-1} dr^2 + r^2 d\Omega_d^2, \quad (25)$$

where μ is a constant and is given by $\mu = 16\pi G_D M / \Omega_d$ in higher dimension, with M as the mass of black hole and Ω_d as the area of a unit d -sphere which is defined by $\Omega_d = 2\pi^{(\frac{d+1}{2})} / \Gamma(\frac{d+1}{2})$.

In the exterior region, therefore, the energy conservation equation (8) takes the special explicit form as follows:

$$p' = -\frac{(p + \rho)[d(d^2 - 1)\mu - 4\Lambda r^{d+1}]}{2[d(d+1)r^d - d(d+1)\mu r - 2\Lambda r^{d+2}]} \quad (26)$$

4 Junction Condition

There are three regions in the gravastar configuration, viz. the interior region, thin shell and exterior region. The shell joins the interior and exterior regions at the junction interface. The metric coefficients are continuous at the shell, however we do not have any confirmation of the continuity of their derivatives. At this juncture we use the condition of Darmois-Israel [45, 46] for calculation of the surface stresses at the junction interface. The intrinsic surface stress-energy tensor S_{ij} takes the form

$$S_{ij} = -\frac{1}{8\pi}(\kappa_{ij} - \delta_{ij}\kappa_{kk}), \quad (27)$$

where $\kappa_{ij} = K_{ij}^+ - K_{ij}^-$, that shows the discontinuity of the extrinsic curvatures or second fundamental forms. Here the signatures $-$ and $+$ describes the interior and exterior boundaries respectively of the gravastar.

Now this extrinsic curvature connect the two sides of the thin shell as

$$K_{ij}^\pm = -n_\nu^\pm \left[\frac{\partial^2 X_\nu}{\partial \xi^1 \partial \xi^j} + \Gamma_{\alpha\beta}^\nu \frac{\partial X^\alpha}{\partial \xi^i} \frac{\partial X^\beta}{\partial \xi^j} \right] |_S, \quad (28)$$

where n_ν^\pm is the unit normals to the surface S can be defined as

$$n_\nu^\pm = \pm \left| g^{\alpha\beta} \frac{\partial f}{\partial X^\alpha} \frac{\partial f}{\partial X^\beta} \right|^{-\frac{1}{2}} \frac{\partial f}{\partial X^\nu}, \quad (29)$$

with $n^\nu n_\nu = 1$.

Following the methodology of Lanczos [47] one can obtain the surface energy-momentum tensor on the thin shell as $S_{ij} = \text{diag}[-\Sigma, p_{\theta_1}, p_{\theta_2}, \dots, p_{\theta_d}]$, where Σ is the surface energy density and $p_{\theta_1} = p_{\theta_2} = \dots = p_{\theta_d} = p_t$ are the surface pressures which respectively can be determined by

$$\Sigma = -\frac{d}{4\pi R} \sqrt{f} \quad (30)$$

$$p_t = -\frac{d-1}{d} \Sigma + \frac{f'}{8\pi\sqrt{f}}. \quad (31)$$

Using the above equations one can obtain

$$\Sigma = -\frac{d}{4\pi R} \left[\sqrt{1 - \frac{\mu}{R^{d-1}} - \frac{2R^2\Lambda}{(d+1)d}} - \sqrt{1 - \frac{16\pi G_D \rho_c}{d(d+1)} R^2 - \frac{2\Lambda R^2}{d+1}} \right] \quad (32)$$

and

$$p_t = \frac{1}{4\pi R} \left[\frac{(d-1) - \frac{(d-1)\mu}{2R^{d-1}} - \frac{2\Lambda R^2}{d+1}}{\sqrt{1 - \frac{\mu}{R^{d-1}} - \frac{2R^2\Lambda}{(d+1)d}}} - \frac{(d-1) - \frac{16\pi G_D \rho_c R^2}{d+1} - \frac{2d\Lambda R^2}{d+1}}{\sqrt{1 - \frac{16\pi G_D \rho_c}{d(d+1)} R^2 - \frac{2\Lambda R^2}{d+1}}} \right]. \quad (33)$$

Now, it is easy to find out the mass m_s of the thin shell as

$$m_s = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})} R^d \Sigma = \Omega_d R^d \left(\sqrt{1 - \frac{16\pi G_D \rho_c R^2}{d(d+1)} - \frac{2\Lambda R^2}{d+1}} - \sqrt{1 - \frac{\mu}{R^{d-1}} - \frac{2R^2\Lambda}{d(d+1)}} \right). \quad (34)$$

Using Eq. (34) we can determine the total mass of the gravastar in terms of the mass of the thin shell in the form

$$\mu = \frac{R^{d+1}}{d(d+1)} [16\pi G_D \rho_c + 2\Lambda(d-1)] + \frac{m_s}{\Omega_d R} \left[2\sqrt{1 - \frac{16\pi G_D \rho_c}{d(d+1)} R^2 - \frac{2\Lambda R^2}{d+1}} - \frac{m_s}{\Omega_d R^d} \right]. \quad (35)$$

5 Physical features of the models

5.1 Energy content

As we consider the thickness of the intermediate shell is very small ($0 < \epsilon \ll 1$), so the phase boundary can be described by the interfaces at $r = R$ and $r = R + \epsilon$ joining the region I and region III respectively, where R describes the phase boundary of the region I.

Using the Eq. (23) we calculate the energy within the shell as

$$E = \int_R^{R+\epsilon} \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})} \right] r^d \rho dr = \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})} \right] \rho_0 k_2 \int_R^{R+\epsilon} \frac{r^{3d}}{|2\Lambda r^2 - d^2 + d|} \quad (36)$$

To solve the above integration (36) over the limit R to $R+\epsilon$, let us form a special differential equation in terms of the parameter $F(r)$ which can be represented as

$$\frac{dF(r)}{dr} = \frac{r^{3d}}{|2\Lambda r^2 - d^2 + d|}. \quad (37)$$

Now using the above consideration we can solve the the integration as follows

$$\int_R^{R+\epsilon} \frac{dF(r)}{dr} dr = [F(r)]_R^{R+\epsilon} = F(R+\epsilon) - F(R) = \epsilon \left(\frac{dF}{dr} \right)_{r=R}. \quad (38)$$

As $\epsilon \ll 1$, so we consider upto the first order term of the Taylor series expansion for the expression in Eq. (38).

Therefore, combining Eqs. (36) and (38) we get

$$E = \left[\frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})} \right] \epsilon \rho_0 k_2 \frac{R^{3d}}{|2\Lambda R^2 - d^2 + d|}, \quad (39)$$

where the square bracket of the integral we have used the factor Ω_d as the area of a unit d -sphere which has already been defined by $\Omega_d = 2\pi^{(\frac{d+1}{2})}/\Gamma(\frac{d+1}{2})$.

From Eq. (39) we observe that the energy within the shell is directly proportional to the thickness of the shell (ϵ).

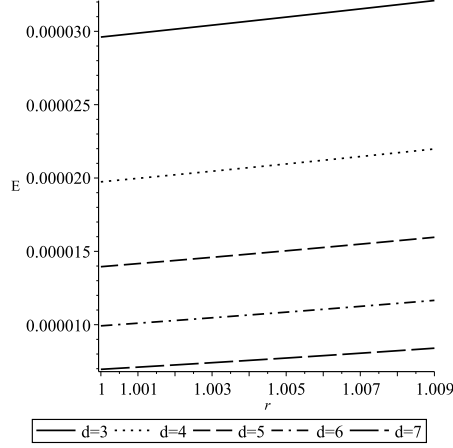


Fig. 2 Variation of the Energy of the shell with r for different dimensions

The variation of E with r for different dimensions is shown in Fig. 2. One can note that the energy is increasing from the interior boundary to the exterior boundary. This is clearly indicating that the shell is getting harder from the interior to exterior boundary, which suggests that the exterior boundary is more dense than the interior boundary as obtained in Fig. 1. The plot also indicates that energy is decreasing as the dimension increases.

5.2 Entropy

Following the prescription by Mazur-Mottola [1] we can use the following equation to calculate the entropy within the shell as,

$$S = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})} \frac{\xi k_B}{\hbar} \sqrt{\frac{\rho_0 k_2}{2\pi}} \int_R^{R+\epsilon} \frac{\epsilon r^{2d}}{\sqrt{(d^2 - d - 2\Lambda r^2)(k_1 + 2(d-1) \ln r - \frac{2\Lambda r^2}{d})}}. \quad (40)$$

The above integration can be solved for small thickness limit ($\epsilon \ll 1$) by using the Taylor series expansion and we obtain

$$S = \frac{2\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d+1}{2})} \frac{\xi k_B}{\hbar} \sqrt{\frac{\rho_0 k_2}{2\pi}} \frac{\epsilon R^{2d}}{\sqrt{(d^2 - d - 2\Lambda R^2)(k_1 + 2(d-1) \ln R - \frac{2\Lambda R^2}{d})}}. \quad (41)$$

It is observed from Eq. (41) that the entropy within the shell is directly proportional with the thickness of the shell. The variation of the entropy (S) with r for different dimensions shown in Fig. 3. and it shows almost similar in nature as the variation of energy obtained in Fig. 2.

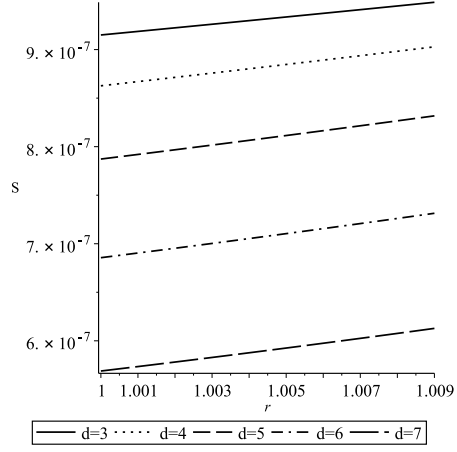


Fig. 3 Variation of the Entropy of the shell with r for different dimensions

5.3 Proper length

Now, the proper length between two interfaces of the shell can be calculated by using the following equation

$$\begin{aligned} \ell &= \int_R^{R+\epsilon} \sqrt{e^\lambda} dr = \int_R^{R+\epsilon} \frac{dr}{\sqrt{k_1 + 2(d-1) \ln r - \frac{2\Lambda r^2}{d}}} \\ &= \frac{\epsilon}{\sqrt{k_1 + 2(d-1) \ln R - \frac{2\Lambda R^2}{d}}}. \end{aligned} \quad (42)$$

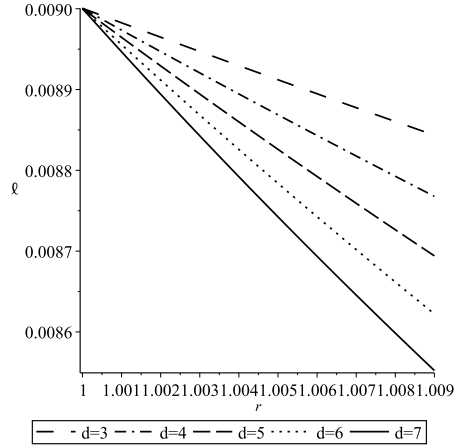


Fig. 4 Variation of the proper length of the shell with the radial coordinate r

5.4 Equation of State

Let us assume that $p_{\theta_1} = p_{\theta_2} = p_{\theta_3} = \dots = p_t = -T$, where T is the surface tension. Therefore, Eqs. (32) and (33) yield

$$T = \omega(R)\Sigma. \quad (43)$$

Thus, the EOS parameter can be found as

$$\omega(R) = - \frac{\left[\frac{\frac{d-1}{d} - \frac{(d-1)\mu}{2dR^{d-1}} - \frac{2\Lambda R^2}{d(d+1)}}{\sqrt{1 - \frac{\mu}{R^{d-1}} - \frac{2R^2\Lambda}{(d+1)d}}} - \frac{\frac{d-1}{d} - \frac{16\pi G_D \rho_c}{d(d+1)} R^2 - \frac{2d\Lambda R^2}{d(d+1)}}{\sqrt{1 - \frac{16\pi G_D \rho_c}{d(d+1)} R^2 - \frac{2\Lambda R^2}{d+1}}} \right]}{\left[\sqrt{1 - \frac{\mu}{R^{d-1}} - \frac{2R^2\Lambda}{(d+1)d}} - \sqrt{1 - \frac{16\pi G_D \rho_c}{d(d+1)} R^2 - \frac{2\Lambda R^2}{d+1}} \right]}. \quad (44)$$

6 Discussion and Conclusion

In the present study of gravastars with higher dimensional manifold and in the presence of cosmological constant, we have considered several aspects of the system. To examine a new model of gravastar in comparison with the type proposed by Mazur-Mottola [1, 2], we are especially searching for its generalization to: (i) the extended D dimensional spacetime from the 4 dimensions, (ii) the Einstein-Maxwell spacetime geometry, and (iii) the cosmological constant. Using the considerations (i) and (ii) we have already obtained a class of solutions [20]. However, in the present work we have used the considerations (i) and (iii) and some interesting results have been explored which can be observed as an alternative to D dimensional Schwarzschild-Tangerlini category black hole [44].

In this section we are discussing some key physical features of the model as follows:

(i) We have obtained several physical parameters, e.g. metric potentials, proper length of the shell, energy, entropy etc. and our results match with the results of Usmani et al. [16] without the cosmological constant in 4 dimensional spacetime. The variations of the parameters as shown in the plots (Figs. 1-4) indicate in favour of the physical acceptability and for the existence of gravastars.

(ii) We have calculated different parameters for three specific regions of the gravastar. All the features of the solutions suggest that the cosmological constant plays an important role for the construction of gravastars.

(iii) From Eqs. (10), (12) and (14) we can observe that the pressure, matter density and the metric potentials are finite at the centre (i.e. $r = 0$) of the gravastar. So the solutions that we have obtained are completely singularity free and also maintain regularity conditions inside the star.

(iv) The variation of the energy and the entropy is shown in Fig. 2 and Fig. 3 respectively, from this figures it can be observed that both energy and entropy is decreasing with the dimensions. This in turn indicates that the shell becomes less compact and the matter density must decrease with dimensions. Exactly the same nature of variation for the matter density within the shell has been observed in Fig. 1.

(v) From Fig. 4 we can observe that the proper length is decreasing with the increase in dimensions, which suggests that the shell become thinner in higher

dimensions. Which is again well justification of the Fig. 1 i.e. as the matter density decreases the energy as well as the entropy will also decreases.

(vi) In comparison with the previous work by Ghosh et al. [20] it can be seen that the presence of cosmological constant for the construction of gravastar in higher dimensions shows almost the similar effects as observed in the presence of charge in the gravastar. This observation therefore indicates that the repulsive nature of the Colombian charge and cosmological constant have the similar role in constructing gravastar.

Finally we can conclude that all the obtained results of the present investigation on gravastar are very much indicative that higher dimensional approach to construct of a gravastar theoretically sound and solutions are physically acceptable. However, a close observation of different profiles and plots we can conclude that the higher dimensional modeling of a gravastar does not indicate any significant difference in nature of the physical parameters from that of the ordinary four dimensional spacetime with or without cosmological constant.

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